Stochastic resonance in delayed two-coupled oscillators without common perturbations

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Stochastic resonance (SR) in a two-coupled oscillator system with time delay is investigated. The system shows multistability of a desynchronized state and two synchronized states with different collective frequencies, which may be interpreted as multistable perception of ambiguous or reversible figures. To model the situations where the two oscillators exist in different environments, periodic signals and noises at their inputs are not uniformly given. SR in an individual oscillator, characterized by the output signal-to-noise ratio, is examined based on numerical simulations. We find that phase shift between the signals at inputs of different oscillators weakens SR, the oscillator with only an input signal does not show SR, and the oscillator with input noise shows SR irrespective of it having an input signal or not. The results have implications in the area of information transmission in biological systems.

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I. INTRODUCTION

Random noise and its effect on physical, chemical, and biological systems have been extensively studied over the past decade [1,2]. The constructive roles of noise are widely being recognized in more nonlinear systems. One important effect of them is stochastic resonance (SR), in which the transmission of a coherent signal is enhanced by additive noise as it passes through a nonlinear system [3–5]. The characteristic signature of SR is the nonmonotonic nature of the signal-to-noise ratio (SNR) as a function of the added noise intensity. SR was first reported in 1981 in the study of glacial period [6–8], and has been explored in various fields theoretically and experimentally.

SR is quite interesting for biological systems [9–11], especially in neurobiological systems [12–18], since SR may provide a mechanism for such systems to detect and process weak signals. In the majority of SR studies on neurobiological systems, coupled oscillator models are often used. Very recently, for example, Kim, Park, and Ryu considered time delay in a coupled oscillator system so that it may be interpreted as multistable perception of ambiguous or reversible figures [19]; later they explored the SR behavior in this delayed system [20], and the result implies that the stochastic switching in multiple perception may be maximized at an optimum noise level. It is worth noting that the coupled models used in many studies on SR, like Kim's model, are usually considered as being in the same environment, in which a common periodic signal and independent uncorrelated noises are added to each element of the coupled systems. Under some circumstances, however, the coupled elements may exist in completely different environments, indicating that the perturbations added to each element may not be common. In such cases, exploring the SR behavior in an individual element is of great help to understand the information transmission and interaction between coupled elements. This idea has been recently considered in investigation of SR in two-coupled threshold elements with input signals shifted in phase, which follows that even if there is a certain phase shift between the signals at inputs of different elements, SR can still be enhanced due to proper coupling [21]. Moreover, this idea has also been extended to the studies of SR in uncoupled systems, i.e., two-parameter chemical systems, in which one control parameter was modulated by a signal and the other by noise [22,23], or one by a signal plus noise and the other by a signal [24].

In this paper, we extend this idea into the above-mentioned coupled oscillator system studied by Kim and coworkers [19,20]. Without external perturbations, this system shows multistability of a desynchronized state and two synchronized states with different collective frequencies. Under the influence of a weak periodic external signal, the system shows a maximum in the SNR at an optimum noise level [20]. Though this model includes the important effect of time delay for biological systems, it does not consider the fact that the individual element may exist in the different environment, which is also major in a real coupled biological system. Here, we choose the simplest coupled case, the two-coupled oscillator system, to examine SR in an individual oscillator as the two oscillators are not perturbed by common signals and noises, so that we could understand the information interaction and transmission between the oscillators. To this end, we select four extreme cases: the first is that each oscillator is perturbed by a signal and noise, but with phase difference between the two signals; the second is that one oscillator is perturbed by a signal and noise, the other by a signal; the third is that one oscillator is perturbed by a signal and noise, the other by noise; the fourth is that one oscillator is perturbed by a signal the other by noise. The perturbation for the first case is symmetrically added, while those for the other three ones are asymmetrically added. Based on numerical simulations, we find that phase difference weakens the SR, the oscillator with only an input signal does not show

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SR, and the oscillator with noise shows SR whether it has an input signal or not.

II. THE MODEL SYSTEM AND ANALYSIS METHODS

A system of two coupled oscillators under study is described by the equation of motion [19,20]

$$d\phi_1(t)/dt=\omega-b\sin[\phi_1(t)]-K/2\{\sin[\phi_1(t)-\phi_1(t-\tau)]+\sin[\phi_1(t)-\phi_2(t-\tau)]\}\quad\text{for oscillator 1,}$$

$$d\phi_2(t)/dt=\omega-b\sin[\phi_2(t)]-K/2\{\sin[\phi_2(t)-\phi_2(t-\tau)]+\sin[\phi_2(t)-\phi_1(t-\tau)]\}\quad\text{for oscillator 2,}\tag{1}$$

where ϕ_1 and ϕ_2 are the phases of the two oscillators, respectively. ω is an intrinsic frequency that is uniformly given to each oscillator. The sum in Eqs. (1) over the two oscillators is describing the time-delayed interaction that depends on their phase difference with delay time τ. See Refs. [19], [20] for details about this model. Here, we choose the parameter values of the system as $\omega=1$, $b=0.5$, $K=1$, and $\tau=3$ with which the system shows the multistability of a desynchronized state and two synchronized states with different collective frequencies.

To investigate SR in the system we apply a weak periodic signal and noise to each oscillator of the system. So Eqs. (1) become

$$d\phi_1(t)/dt=\omega-b\sin[\phi_1(t)]-K/2\{\sin[\phi_1(t)-\phi_1(t-\tau)]+\sin[\phi_1(t)-\phi_2(t-\tau)]\}+\varepsilon_1\sin(\Omega_1 t+\theta_1)+\xi_1(t)\quad\text{for oscillator 1}$$

$$d\phi_2(t)/dt=\omega-b\sin[\phi_2(t)]-K/2\{\sin[\phi_2(t)-\phi_2(t-\tau)]+\sin[\phi_2(t)-\phi_1(t-\tau)]\}+\varepsilon_2\sin(\Omega_2 t+\theta_2)+\xi_2(t)\quad\text{for oscillator 2,}\tag{2}$$

where ε_1, Ω_1, θ_1 and ε_2, Ω_2, θ_2 are the amplitude, frequency, and initial phase of the signals for the two oscillators. $\xi_1(t)$ and $\xi_2(t)$ are the Gaussian white noises characterized by

$$\langle\xi_1(t)\rangle=0,\ \langle\xi_2(t)\rangle=0,$$

$$\langle\xi_1(t)\xi_1(t')\rangle=2D_1\delta(t-t'),\ \langle\xi_2(t)\xi_2(t')\rangle=2D_2\delta(t-t')$$

in which D_1 and D_2 measures intensity of the additive noise. Here, we choose the signal with $\varepsilon_1=\varepsilon_2\leqslant 0.2$ throughout so that it could not induce the transition among the multistable states in the absence of noise. To quantify the SR effect, 16 384 points of $d\phi(t)/dt$ at intervals of 1 sec are used to obtain frequency spectra by fast Fourier transformation. SNR is defined as the ratio of the height of the spectrum of the output signal at frequency Ω to the average amplitude of the background noise spectrum in the vicinity of Ω [25]. In this work Ω is selected to be the frequency of additive signal. Each plot of SNR vs noise intensity is obtained from 30 independent runs.

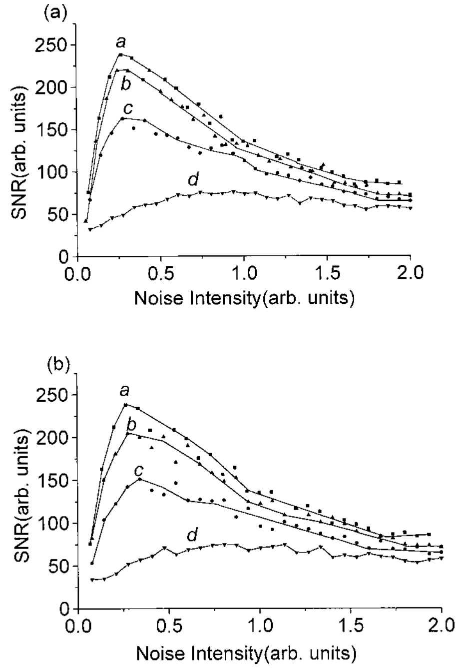

FIG. 1. $s_1n_1s_2n_2$: SNR as evaluated from the frequency spectra vs noise intensity for $d\phi_1/dt$ (a) and $d\phi_2/dt$ (b) with $D_1=D_2$, $\varepsilon_1=\varepsilon_2=0.2$, $\Omega_1=\Omega_2=0.01/2\pi$, and $\theta_1=0$, $\theta_2=0$ (a), $\theta_2=\pi/4$ (b), $\theta_2=\pi/2$ (c), and $\theta_2=\pi$ (d). The two axes in each plot of this paper have arbitrary units. The solid lines are merely to guide the eyes.

III. NUMERICAL RESULTS AND THEIR DISCUSSION

A. Oscillators symmetrically perturbed with input signal shifted in phase

Consider the two-coupled oscillators to form a spatially extended system. There exists only one periodic signal source that can provide signal to the oscillators. Besides the signal source, each oscillator is perturbed by its internal noise as well. Owing to a finite propagation time of signal, the signals into the two distant oscillators may be shifted in phase. Furthermore, due to different propagation distances from the signal source to the two oscillators, the signals can also be different in strength. Here, we investigate the effect caused only by phase difference. Thus, we suppose that the intensities of the signals into the two oscillators are identical, and so are the noise intensities for them. In other words, oscillators are driven by periodic signals that have the same amplitude and frequency, but different phase at the input of each oscillator, and by independent noises of the same intensity. This case being realized in Eqs. (2), we can set $D_1=D_2$, $\varepsilon_1=\varepsilon_2=0.2$, $\Omega_1=\Omega_2=0.01/2\pi$, $\theta_1=0$, and $\theta_2-\theta_1\neq 2k\pi$, where k is an integer. Since both oscillators are added by a signal and noise, we call such case $s_1n_1s_2n_2$. Figure 1(a) shows the plots of SNR of $d\phi_1(t)/dt$ against noise intensity for the phase difference 0 (a), $\pi/4$ (b), $\pi/2$ (c), and π (d), respectively. It is obvious that with the increment of phase difference from $0\sim\pi$ SR monotonically weakens. Interestingly, as the phase difference is equal to π, i.e., completely reverse phase between the signals, the SR strength decreases to the minimum. This phenomenon seems to indicate a resonancelike behavior between the SR strength and the phase difference; as the phase difference is equal to zero the SR strength reaches the maximum. Figure 1(b) shows the plots of SNR of $d\phi_2(t)/dt$ against noise intensity, exhibiting the same resonancelike behavior as those of $d\phi_1(t)/dt$. It should be noted that Fig. 1(a) is very similar to Fig. 1(b) both in shape and size. This implies that the phase difference cannot break the symmetry of the original coupled system, i.e., the two oscillators still have identical response to the noise of the same intensity. This symmetry is maintained because of the symmetric perturbations suppressed onto them. In the following section, however, we will see that this symmetry can be broken by asymmetrically added perturbations.

B. Oscillators with perturbations asymmetrically added

In this section we consider a few more complex situations than the above-mentioned case, such as that owing to the block of signal propagation one oscillator may receive much weaker signal than that the other oscillator receives, and that owing to lying in a higher-temperature environment one oscillator's internal noise may be stronger than that of the other oscillator, etc. To model these situations we select three extreme cases, where the perturbations onto the two oscillators are asymmetrically added.

The first case is that one oscillator is perturbed by a signal and noise and the other only by a signal. So we set $D_1\neq 0$, $D_2=0$, $\varepsilon_1=\varepsilon_2=0.2$, and $\Omega_1=\Omega_2=0.01/2\pi$ in Eqs. (2). We call this case $s_1n_1s_2$. As in the above investigation of $s_1n_1s_2n_2$, Fig. 2(a) and (b) show SNR vs noise intensity D_1 for $d\phi_1(t)/dt$ and $d\phi_2(t)/dt$, respectively, for the four typical initial phase shifts. The curves in Fig. 2(a) all show the obvious characteristic signature of SR, while those in Fig. 2(b) show only an increase of SNR to a constant with the increment of noise intensity. The latter is understandable if we note that in Eqs. (2), ϕ_1, which introduces noise into $d\phi_2(t)/dt$, is in the term of the sinusoidal function, so the practical noise intensity for $d\phi_2(t)/dt$ cannot increase continuously with the increment of the noise intensity of oscillator 1. The effect of phase difference between Ω_1 and Ω_2 can also be obviously seen in Fig. 2. Like $s_1n_1s_2n_2$, SR for $d\phi_1(t)/dt$ is weakened by phase difference, indicating the effect of phase difference still works in this asymmetric case. The SNR curve for $d\phi_2(t)/dt$ is weakened as well by phase difference, especially in the forepart of the curve. Another point of interest is that the transmission of the power of noise from oscillator 1 to oscillator 2 leads to oscillator 1 having less insensitivity than that in $s_1n_1s_2n_2$ to the noise of the same intensity. This point can be seen from the fact that the peaks of the SRs in Fig. 2(a) become wider, and their maximum effects are shifted to higher noise intensity than the corresponding SRs in Fig. 1(a).

As seen above, the oscillator added with a signal and noise can transmit its noise information to the coupled oscillator with only signal input. And with the help of the transmitted noise information the latter can increase its SNR to a constant. The question now is what about the signal transmission. To this end, we set the second extreme case as that one oscillator is perturbed by noise and the other by a signal plus noise. For this case we set $D_1=D_2$, $\varepsilon_1=0$, $\varepsilon_2=0.2$, and $\Omega_2=0.01/2\pi$ (case 1), $0.08/2\pi$ (case 2), and $0.1/2\pi$ (case 3) in Eqs. (2). We call such case $s_2n_2n_1$. The plots of SNR as a function of noise intensity for oscillator 1 with the three signal frequencies are shown in Fig. 3(a) exhibiting a peak at an optimum noise level; the characteristic signature of SR. It follows that the oscillator 2 can as well transmit its signal information into oscillator 1, and the transmitted signal information can be maximized at an optimum noise level. It should be noted that the SR of oscillator 1 is much weaker than that in $s_1n_1s_2n_2$, indicating that the signal transmitted to oscillator 1 is very weak. Another point worth noting is that the three SRs of oscillator 1 are obviously different in strength. In other words, different frequency signals from oscillator 2 can transmit signals of different intensities to oscillator 1, indicating a selective mechanism for signal transmission. The plots of SNR against noise intensity for oscillator 2 with the three frequencies are shown in Fig. 3(b). They all show SR behavior, however, quite different in strength. Comparing the orders of SR intensity in Fig. 3(a) and (b), we can readily see that the above selective mechanism is closely related to the abilities of oscillator 2 to receive signals of different frequencies. In addition, for the case of $\Omega_2=0.01/2\pi$ [see Fig. 3(b), curve a] oscillator 2 shows a weaker SR than the corresponding one in $s_1n_1s_2n_2$

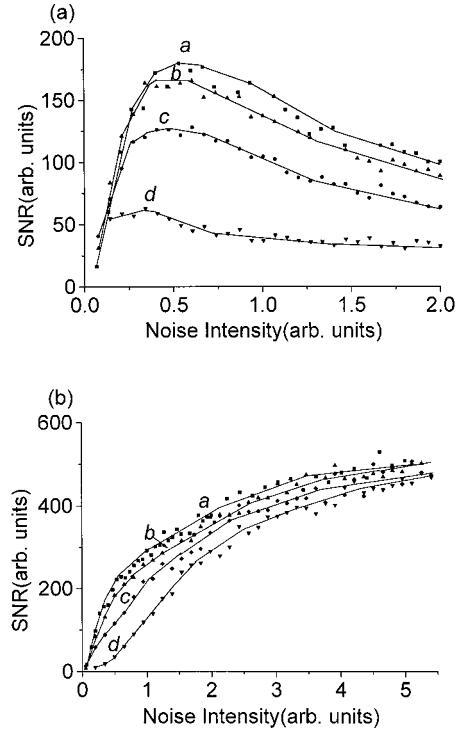

FIG. 2. $s_1n_1s_2$: SNR vs noise intensity D_1 for $d\phi_1/dt$ (a) and $d\phi_2/dt$ (b) with $D_1\neq 0$, $D_2=0$, $\varepsilon_1=\varepsilon_2=0.2$, $\Omega_1=\Omega_2=0.01/2\pi$, and $\theta_1=0$, $\theta_2=0$ (*a*), $\theta_2=\pi/4$ (*b*), $\theta_2=\pi/2$ (*c*), and $\theta_2=\pi$ (*d*). The solid lines are merely to guide the eyes.

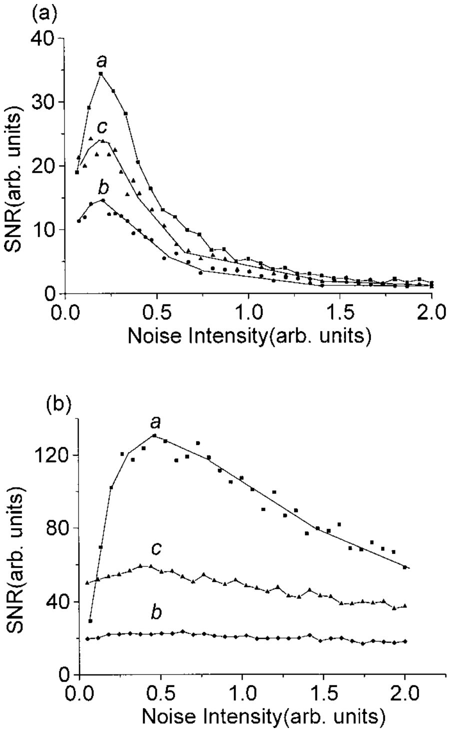

FIG. 3. $s_2n_2n_1$: SNR vs noise intensity for $d\phi_1/dt$ (a) and $d\phi_2/dt$ (b) with $D_1=D_2$, $\varepsilon_1=0$, and $\varepsilon_2=0.2$, and $\Omega_2=0.01/2\pi$ (*a*), $\Omega_2=0.08/2\pi$ (*b*), and $\Omega_2=0.1/2\pi$ (*c*). The solid lines are merely to guide the eyes.

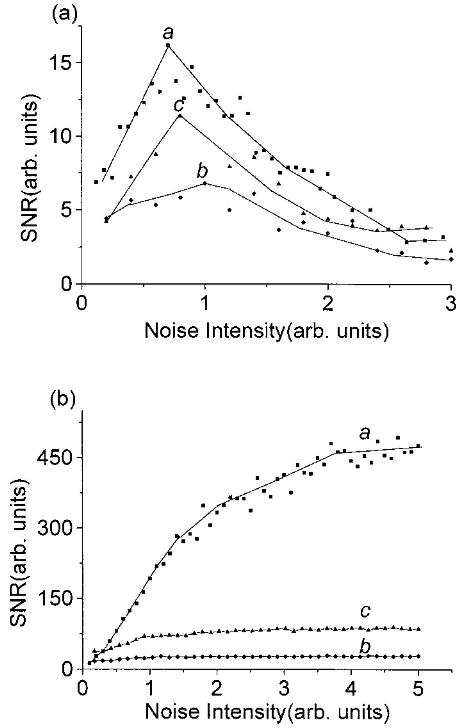

FIG. 4. s_2n_1: SNR vs noise intensity D_1 for $d\phi_1/dt$ (a) and $d\phi_2/dt$ (b) with $D_1\neq 0$, $D_2=0$, $\varepsilon_1=0$, and $\varepsilon_2=0.2$, and $\Omega_2=0.01/2\pi$ (*a*), $\Omega_2=0.08/2\pi$ (*b*), and $\Omega_2=0.1/2\pi$ (*c*). The solid lines are merely to guide the eyes.

[see Fig. 1(b), curve *a*], because in $s_2n_2n_1$ oscillator 2 only plays a role of transmitting its signal to oscillator 1, while in $s_1n_1s_2n_2$ oscillator 2 also plays a second role of receiving signal from oscillator 1.

When deleting n_2 in $s_2n_2n_1$, it becomes the simplest way that may produce SR for a two-coupled oscillator system, and also the last case s_2n_1 studied here. In this case we want to see more clearly the transmission of signal and noise between the two oscillators. Like $s_1n_1s_2$, the oscillator 1 transmits its noise information to oscillator 2, and since the noise into oscillator 2 comes from the sinusoidal function, $d\phi_2(t)/dt$ only shows an increase of SNR to a constant with the increment of noise intensity, as shown in Fig. 4(b). It should be noted that the SNR curves in Fig. 4(b) corresponding to different Ω_2, just like Fig. 3(b), are quite different in strength as well, and the order of the SNR intensity is in coincidence with that of the SR intensity in Fig. 3(b). For oscillator 1, like that in $s_2n_2n_1$, $d\phi_1(t)/dt$ shows SR since oscillator 2 transmits its signal to oscillator 1. In addition, the selective mechanism of signal transmission still exists in this case, as shown in Fig. 4(a). Compared with the SRs in Fig. 3(a), however, the peaks of the corresponding ones in Fig. 4(a) become wider, and their maximum effects are shifted to higher noise intensity. This fact may be understandable if we note that in s_2n_1 oscillator 1 plays only the role of transmitting its noise to oscillator 2, while in $s_2n_2n_1$ oscillator 1 also plays a second role of receiving noise from oscillator 2. Thus, just like that discussed in $s_1n_1s_2$, oscillator 1 shows less insensitivity than that in $s_2n_2n_1$ to the noise of the same intensity.

Based on the above results, we conclude that the phase difference of the signals in the two oscillators weakens SR (see oscillators 1 and 2 in $s_1n_1s_2n_2$, and oscillator 1 in $s_1n_1s_2$), the oscillator with only signal input does not show SR (see oscillator 2 in $s_1n_1s_2$ and oscillator 2 in s_2n_1), and the oscillator with noise shows SR irrespective of it having signal or not (see oscillator 1 in $s_2n_2n_1$ and oscillator 1 in s_2n_1). In addition, a phenomenon of frequency selection is also observed (see oscillator 1 in $s_2n_2n_1$ and oscillator 1 in s_2n_1). These results have implications in the area of information transmission in coupled-element biological systems. For signals transmitted into these elements, phase shifts between these signals may play a major role, such as that different phase shifts cause the system's different sensitivities to noise indicating the directivity of signal transmission. For example, considering the situation where the above-studied two-coupled oscillator system has more than one signal source, the question is which sources can provide more efficiently their signals to the oscillators through SR mechanism. According to one result presented here, the less the phase difference between the signals at the input of each oscillator is, the more is the efficiency. So our answer to the above question is that those signal sources, which have such positions that the difference of their distances from oscillators is about the integer times the wave length of the periodic signal, may more efficiently provide their signals to the oscillators. On the other hand, though coupled elements being in different environments may cause different perturbations suppressed onto each element, they may transmit signals and noises to one another so as to share information. However, there may exist a selective mechanism for different frequency signals, indicating a filtering effect in the process of more than one signal transmissions. It would be interesting if these results and phenomena could be tested and observed in the real systems.

IV. CONCLUSION

Each element of coupled models, studied previously in the investigations of SR, is often perturbed by a common periodic signal and independent uncorrelated noises, while in this work we investigate SR in individual oscillators in a delayed two-coupled oscillator system from Kim's model [20], where perturbations including periodic signals and random noises are not uniformly given to the two oscillators. It is found that the phase difference of the signals in the two oscillators weakens SR, the oscillator with only signal input does not show SR, and the oscillator with noise shows SR irrespective of it having a signal or not. These results imply that they may transmit, via coupling, individual signals and noises to each other so as to share information. Though these results are obtained in the two-coupled case of Kim's model, we think they may be extended to the multicoupled systems.

At the end of this paper, we stress that it is significant to consider elements in different environments in investigations of real perturbed coupled systems, since many living and nonliving systems suffer from many kinds of periodic and stochastic fluctuations simultaneously [22,26–28]. We wish this work could attract more attention of researchers in this field to promote to extend this idea into their researches.

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